Math and Music Worksheet

1. Starting with a note having a frequency of 110 Hz, find the frequencies of the notes that are one, two, three and four octaves higher.

2. Starting with a note having a frequency of 880 Hz, find the frequencies of the notes that are one, two, three and four octaves lower.

3. Using the keyboard below, write the notes in the G major scale.

4. Using the keyboard above, write the notes in the A major scale.

5. Find the frequencies of the 12 notes of the scale that starts at A above middle C; this A has a frequency of 440 Hz.

6. Assume middle C has a frequency of 256 cycles per second. Find the frequencies of the 12 notes of the scale that starts at middle C.

7. The circle of fifths is generated by starting at a particular note and then going up an interval of a fifth (seven half-steps). For example, starting at middle A, a circle of fifths includes the notes A → E → B' → F' → C'' → G'' → D''' → A'''', → F''' → C''' → G''' → D'''' → A'''', where () denotes a higher octave. Frequency can also go in fifths by using the formula \((3/2)^n\) where \(x\) is the original note and \(y\) is the fifth note above the original.
   a. Follow the circle of fifths starting with middle A at 440 Hz and obtain the frequency of C five octaves above.
   b. Now find the values of the frequencies for each octave below until you return to middle C, show that this leads to middle C having a frequency of 264 Hz approximately.
   c. Starting with A (440 Hz), alternate between finding the fifth above and the octave below until you get a value for middle C. Is it the same value from part (b)?

8. Starting at middle A with a frequency of 440 Hz, find the frequency of the following notes:
   a. Four half steps (a major third) above middle A
   b. A fifth above (seven half steps) middle A
   c. An octave and a third above middle A
9. The circle of fourths is generated by starting at a particular note and then going up an interval of a fifth (seven half-steps). For example, starting at middle A, a circle of fifths includes the notes A → D → G → C′ → F′ → A♯″ → D♯″ → G♯″ → C♯″ → F♯″ → B♯″ → E♯″ → A♯″″, where (’') denotes a higher octave. Frequency can also go in fourths by using the formula \((\frac{4}{3})^x = y\) where \(x\) is the original note and \(y\) is the fourth note above the original.
   a. Follow the circle of fourths starting with middle A at 440 Hz and obtain the frequency of C one octave above.
   b. Now find the values of the frequencies for each octave below until you return to middle C, show that this leads to middle C having a frequency of 261 Hz approximately.

10. The F♯ major pentatonic scale consists of the notes F♯, G♯, A♯, C♯, and D♯. It corresponds to the five blacks keys in one octave on a keyboard. Write a short paper on the use of the pentatonic scale in different cultural music, such as Celtic, Chinese, African, Korean, etc.

11. In the 1930's, music theorist Joseph Yasser published a book, *Theory of Evolving Tonality*, exploring the idea of using a 19-tone scale instead of 12-tone scale. Write a short paper discussing his ideas, the connection with the Fibonacci numbers, and possible reasons why the 19-tone scale never caught on.

12. In 1918, a Soviet scientist named Leon Theremin invented one of the world's first electronic musical instrument. The “Theremin” as it became known was unlike any other instrument, it had no strings, pipes, or keys, in fact, the player didn't even have to touch the instrument to play it. For those who have seen the Led Zeppelin movie, *The Song Remains the Same*, Jimmy Page plays one during the song “Whole Lotta Love.” Write a short paper on the history of this instrument include its significance to Robert Moog and his Moog music synthesizer. You may want to watch the documentary DVD, *Theremin - An Electronic Odyssey*.

13. John Cage (1912-1992) began to use the *I Ching* in the composition of his music in order to introduce an element of chance over which he would have no control. He used it, for example, in the *Music of Changes* for solo piano in 1951, to determine which notes should be used and when they should sound.

14. What is twelve-tone music or dodecaphony? What is the fundamental rule of 12-tone music?

15. Who is Igor Stravinsky (1882-1971)? Write a review of his twelve-tone music.

16. Johann Sebastian Bach wrote many canons and experimented with all sorts of different forms. One very interesting canon is his canon for two violins, also called the “crab canon.” What is interesting about this musical piece? Why do you think it is known as the crab canon?

Lewinter & Widulski
17. Who is Arnold Schönberg (1874-1951)? What does he have to do with twelve tone music? What does he have to do with crab canon?

18. Who are Harry Partch (1901-1974) and Béla Bartók (1881 - 1945)?

19. The Mozart Effect. One of the most publicized connections between music and academics (including mathematics) is the “Mozart effect”. The idea originated from a letter by Frances Rauscher, Gordon Shaw, and Katherine Ky published in the October 14, 1993 issue of Nature. These three University of California at Irvine researchers summarized the findings of an experiment they conducted on their students. The students who listened to Mozart showed an increase in their IQ scores. Their results lead to the claim that exposure to certain types of music, especially classical music, early on in life, can lead to improved performance on test scores, including tests of spatial visualization, abstract reasoning, and so on. Use the Internet to research the Mozart effect and summarize your finding in a three-page paper.

20. What is fractal music? Write a short paper on this subject and identify some of the present day composers.