Egyptian Geometry Worksheet

We can summarize Egyptian geometry by stating that which has been found consists of a number of problems in which the inclination of a line and a plane was determined, or volumes and areas of mathematical figures were computed. The solutions of these problems proceeded according to definite arithmetical instructions, some of which are correct and some of which are not. The calculations of the area of a rectangle, a triangle, and a trapezoid were all correct. There is no record of statements of general theorems or of proofs; the chief concern of the Egyptian seemed to be to obtain a useful result.

A. Area of the circle by the Egyptian method.

"Take from its diameter a one-ninth part. The result shall form the side of a square whose area is equal to that of the circle."

a. Egyptian Method:

i. Formula:

The side of the square is $d - \frac{1}{9}d = \frac{8}{9}d$.

The area of the square and, thus, the circle is $A = \left(\frac{8}{9}d\right)^2 = \frac{64}{81}d^2$.

ii. Example:

If d = 9, the area of the circle is

$$A = \frac{64}{81}d^2 = \frac{64}{81}(9)^2 = \frac{64}{81} \times 81 = 64$$

b. Correct Method:

i. Formula:
$$A = \mathbf{p} \cdot r^2 = \mathbf{p} \cdot \left(\frac{d}{2}\right)^2 = \frac{\mathbf{p}}{4}d^2$$

ii. Example: If
$$d = 9$$
, then $A = \frac{p}{4}(9)^2 = \frac{81}{4}p \approx 63.62$

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Homework Exercises

Find the area for each circle with the following diameters. Use both methods as discussed in class.



B. The Volume of a Pyramid

Surely, the pyramids are visual proof of the Egyptian proficiency in mathematics. Suppose we have a square ABCD whose side is length b, that lies in a horizontal plane, i.e., the ground, and that a perpendicular, of length h, is erected in the center of the square, then the resulting figure is a square pyramid.



The Egyptians knew that the formula for a pyramid is $V = \frac{1}{3} \times b^2 \times h$ where *b* is the length of the side of the square and *h* is the height. Use this formula to solve the following exercises.

Homework Exercises

Find the volume for each pyramid with the following bases and heights.

1.
$$b = 10, h = 7$$

- 2. b = 3, h = 4
- 3. b = 6, h = 10
- 4. b = 5, h = 15
- 5. b = 14, h = 20

C. The Volume of a Truncated Pyramid

A truncated pyramid, called a *frustum*, is a pyramid with its top cut off or truncated. Let the bottom be a square with side b_1 and let the top be a square with side b_2 . If the height is *h* then the formula for the volume is $V = \frac{1}{3}h(b_1^2 + b_1b_2 + b_2^2)$.



Homework Exercises

Find the volume for each frustum with the following bases and heights.

- 1. $b_1 = 10, b_2 = 4$, and h = 7
- 2. $b_1 = 3, b_2 = 2, \text{ and } h = 9$
- 3. $b_1 = 9$, $b_2 = 6$, and h = 5
- 4. $b_1 = 7$, $b_2 = 4$, and h = 10
- 5. $b_1 = 12, b_2 = 3$, and h = 20